

Fast Training Algorithm for Genetic Fuzzy Controllers and application to an Inverted Pendulum with Free Cart

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Abstract. The classical control theory cannot be applied in those systems whose complexity is too high to be analytically modeled. In these cases, mathematical methods with more degrees of freedom are used because they provide better adaptation. One method widely used in control problems is the Fuzzy Inferencing Systems. However, the process of calibration of the parameters required may involve a high computational cost. Among them, Genetic Algorithms have demonstrated great convergence towards ideal solutions. As the dimensions of the control problem (input features) increase, the optimization process requires much more time. Therefore, the present work proposes a gradual search and parameter update criteria for Genetic Fuzzy Controllers because it improves several orders of magnitude in the processing time. The algorithm developed has been applied to the control problem of the Inverted Pendulum with Free Cart. The results obtained demonstrate an effective parameter calibration in seconds, while the traditional method of tuning for the same problem takes more than 2 hours. Currently, many of the mechanical systems of the different industries undergo sudden changes in their properties during use, therefore an instant effective recalibration of the controllers is necessary. This method allows fast adaptation and also guarantees the same performance in the control process.

Keywords: Fast Tuning, Chained Genetic Algorithms, Fuzzy Inferencing System, Symmetric Mechanical Systems, Gradual Update

1 Introduction

1.1 Problem

The use of a single Genetic Algorithm (GA) that optimizes the parameters of an entire Fuzzy Inference System (FIS) in a problem with several degrees of freedom is computationally expensive.

1.2 Main Purpose

Offer a substantially faster training method for the FIS of a control problem with symmetry, while proving high precision in the testing.

1.3 Approach

Instead of applying a single GA for the exploratory search of the parameters for the control, the following steps are proposed:

Divide the problem into small independent blocks (workspace regions of the FIS).

Apply different chained GAs to some of the blocks (strategically chosen) to optimize these regions.

Finally, extrapolate the information to the remaining blocks (symmetrically equivalent) without the use of GAs.

In the end, a fully trained FIS is achieved in a faster and elegant way rather than using “brute force” in the optimization.

1.4 Nature of the Case Study

The case chosen to benchmark this technique is the Inverted Pendulum with Free Cart (IPFC). The System is composed of a homogeneous bar with a mass M_2 and a length L_2 (known moment of inertia I_2), joined with a linkage that allows its rotation to the top of the cart. The free cart, of mass M_1 , glides without friction along a horizontal track. A unique actuator exerts a horizontal force, F , in the center of gravity of the cart, G_1 .

The IPFC problem has 2 degrees of freedom and thus it should have 2 actuators. Nevertheless, the configuration considered includes a single actuator to increase the complexity of the problem. Thus, proving the efficiency of the algorithm developed in the control of highly unstable mechanical systems.

Three input variables have been chosen to control the system, $\theta, \ddot{\theta}, \ddot{x}$ angle of the rod, angular acceleration and linear acceleration of the base of the cart respectively. There is no control over $\dot{\theta}$ (angular velocity of the rod) or x (linear displacement of the cart). As said, F (horizontal force applied to the cart) will be the control variable on the actuator. Thus, the problem is 4-dimensional with 3 inputs and 1 output (see Fig. 1).

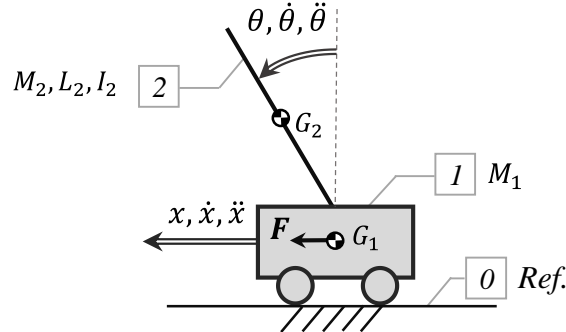


Fig. 1. Schematics of the Inverted Pendulum with Free Cart (IPFC).

For the given configuration, the differential equations of the motion of the IPFC with no friction are defined as,

$$F = (M_1 + M_2)\ddot{x} + M_2 \frac{L}{2} \ddot{\theta} \cos(\theta) - M_2 \frac{L}{2} \dot{\theta}^2 \sin(\theta) \quad (1)$$

$$\left(I_{2_G} + M_2 \frac{L^2}{4} \right) \ddot{\theta} = M_2 g \frac{L}{2} \sin(\theta) - M_2 \frac{L}{2} \ddot{x} \cos(\theta) \quad (2)$$

2 Related Work

For the control of this nonlinear system (IPFC), many and very diverse strategies have been used previously. In [1] such objective is obtained with the Lyapunov function and applying LaSalle's invariance theorem.

In [2] a network of the Fuzzy Logic Controllers (FLCs) with different weights and a GA for the tuning is used. The control of the double inverted pendulum is also an interesting problem, similar to the one considered in this study, where a Genetic Fuzzy System proves to be a good solution ([3]).

A comparative study of the LQR and PID control strategies is introduced in [4]. In [5] the passivity properties of Lagrangian and Hamiltonian systems are used to find the best static feedback controller.

Other authors have used GAs for the optimization of the parameters for the FIS of the inverted pendulum and cart system but at a high computational cost, [6]. Particle Swarm Optimization (PSO) algorithms or Ant Colony System optimization techniques are viable substitutes to replace the use of GA for the tuning of the PID or the LQR ([7], [8]). Alternative inferencing rules considering the Takagi-Sugeno were introduced in [9] for the control of the inverted pendulum.

Nevertheless, the training time of the inferencing rules for those methodologies is still high. There are already algorithms which use neuro-fuzzy logic controllers with the aim to diminish such computational cost ([10]).

3 Theoretical Background

The concept of fuzzy refers to a scale of values between 0 and 1. Where the analyzed variable can belong to more than one set. In this way, the transition from one group of values to another is blurred. Thus, a gradation in the contours that limit the membership of each set is obtained. In short, it is intended to resolve the sudden changes that crisp sets imply.

In engineering control systems, a fuzzy data processing approach significantly improves the accuracy. That is, by increasing the degrees of freedom, the control surface (or hypersurface for more than 2 dimensions) becomes smooth, with no sudden jumps. A great ability to adjust is achieved regardless of the problem considered.

In the vast majority of aerospace systems these precision and adaptation properties are essential, given the complex nature of the infrastructure. However, this technique presents a clear drawback when compared to the classical control theory; it requires the optimization of a greater amount of values.

Therefore, an efficient optimization method is needed to perform an exploratory search until the closest combination of parameters is found. Genetic Algorithms (GAs) have been one of the biggest bets for the search of these parameters. This algorithm is based on the principles of natural evolution. A group of chained functions perform the basic processes that are observed in the biological mechanisms for development; selection, mutation, crossover and elitism. In each generation / iteration, the individuals (chromosomes) of the population change, progressively improving their fitness. At all times, chromosomes are evaluated with a fitness function that determines how valid the individual is. A certain degree of randomness is also incorporated in the evolutionary process. This is done to consider alternative solutions, which although in the initial generations are not very adequate, in the future they will lead to better results.

The IPFC problem involves an exploration of continuous values. Therefore, the training process using a single GA (in this study referred to as the traditional approach) can take a long time if the number of parameters to be optimized is high.

Therefore, the authors have focused their work on improving the Genetic Fuzzy (GF) system so that the computational cost for training is significantly lower than the one required in the traditional approach.

One of the techniques developed is the Modified Mamdani ([11]) that allows solving the problem of 3D fuzzy fitting in seconds (see Fig. 2). This simulates the adaptation of a Neural Network with step functions by making individual approximations in each section of the surface.

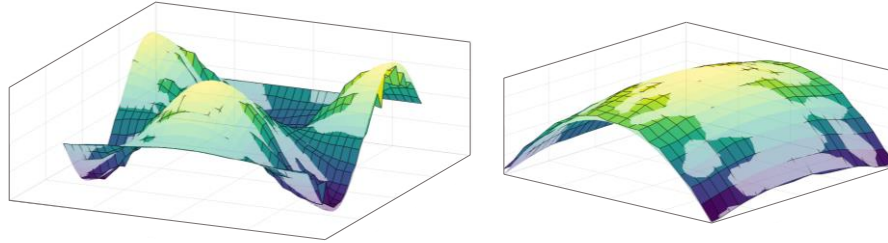


Fig. 2. 3D Genetic Fuzzy Fitting with a Modified Mamdani Approach (real solution in transparent, and approximation in color).

The technique proposed in this paper seeks also this property of fast training. The physical properties of many industrial systems are subject to sudden modifications. Thus, an immediate recalibration process should be used in order to minimize the loss of efficiency. The recalibration and maintenance operations of the tools are still a big problem in many automated manufacturing processes.

For this preliminary development, the primary goal was to obtain a successful fast learning algorithm. In the next stage, this fast adaption skill will be incorporated in the controller of an IPFC prototype.

The system will have a health monitoring systems integrated (accelerometers and gyroscopes will be used for the measurements), which will evaluate constantly the efficiency of the controller. When the IPFC shows a decrease in performance (Identification Procedure) the fast learning algorithm will recalibrate automatically the coefficients of the FIS to obtain a tuned system (Self Adaption Procedure) (Fig. 3).

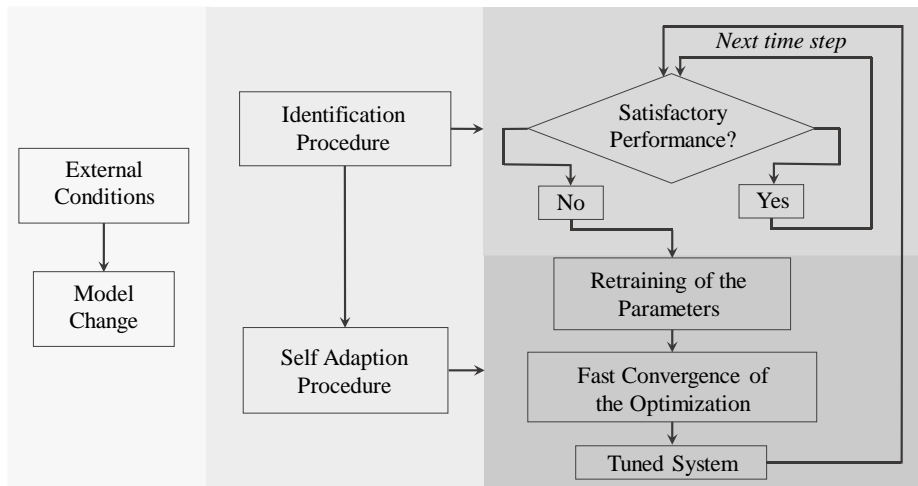


Fig. 3. Block Diagram for Self Adaptation of the IPFC.

4 Methodology

In the first place, a preprocessing of the variables has been carried out, being standardized with values between 0 and 1. As initial input, it has been decided to divide the 4 variables into 5 equally spaced regions. Identifying the letters L, H, Z, N and P as Low, High, Zero, Negative and Positive respectively.

In the proposed algorithm, membership functions with symmetrical triangular shape have been used. The two functions of the ends are therefore divided in half, generating right triangles. The problem is thus reduced to the choice of the values (5 possible) of each combination in the three-dimensional mesh of the if-then-rules.

Fig. 4 shows the structure or 3D tower of if-then rules along with the three input variables and their associated membership functions. The form of the membership functions of the output variable is not represented in Fig. 4, however, it is the same as the other three.

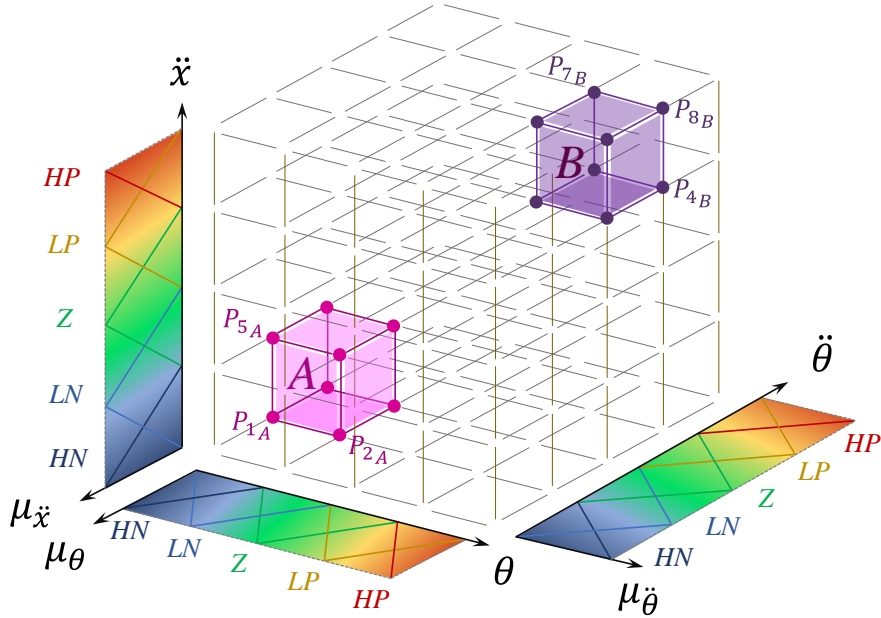


Fig. 4. Tower or grit of if-then rules representing all the possible combinations of the fuzzy sets from the input variables.

Every point in the 3D grit represents an if-then rule. Thus, the total number of if-then rules is 5^3 , 125 parameters for the optimization problem.

In the traditional approach (single GA for the optimization of all the parameters) if the form of the memberships is not defined, the number of degrees of freedom due to the membership functions should also be counted. Since there are 4 variables, each

with 5 membership functions, and each membership function has 3 edges, there are 60 degrees of freedom. Thus, making a total of 185 values for the optimization problem.

In a normal computer of basic 8GB CPU this traditional approach requires more than 2 hours of computation. On the other hand, the training of the algorithm presented requires less than 10 seconds.

For the obtention of the membership value for the target variable the geometric mean is calculated using the three variables' memberships. Then, a defuzzification process is carried out. The final value is proportional to the areas enclosed under the membership functions of the output force.

The fitness function used is such that it seeks to maintain vertical the pendulum ($\theta \rightarrow 0$), minimizing its angular acceleration ($\ddot{\theta} \rightarrow 0$) and linear acceleration ($\ddot{x} \rightarrow 0$).

Fig. 4 also represents with different colors two subsets (A and B) that contain eight if-then rules each. Due to the symmetry of the problem, these subsets are complementary, such that each point of the subset A has its exact complementary inside B. As defined below.

$$P_{1A} \equiv \overline{P_{8B}} \quad (3)$$

$$P_{2A} \equiv \overline{P_{7B}} \quad (4)$$

$$P_{5A} \equiv \overline{P_{4B}} \quad (5)$$

For the optimization problem, several recursive functions are used. The ultimate goal is to find which of the membership functions from the output variable, F , corresponds to each of the points in the 3D lattice.

The tuning is then performed inside each subset. It starts from one corner of the 3D lattice, where a single GA is applied to search for the eight parameters of the cell in the lattice. After the generations converge to the best chromosome, the corners of that subset are updated with the values of the genes. The first exploration process finishes, and a new GA is applied in the following subset that lies next to the current one. The only difference is that the length of the chromosomes is reduced to the half. Thus, convergence is much faster.

This process is repeated for each subset of the 3D tower, updating the inferencing rules as the chained GAs provide the best solution for each subset.

The optimization is not the same for all the cells in the workspace, the dimensionality of the problem plays a key role to determine the most efficient updating protocol.

Additionally, as it has been pointed out before, due to the symmetry, the computational time of the problem can be reduced to the half. But in fact, this is not the condition that improves significantly the computational cost. The key to minimize the processing time is the discretization of the workspace into subsets, and the gradual update of the lattice by using several GAs that pass the information forward, creating a chain. In the end, the exploratory problem is reduced tremendously, proving that is more efficient the application of many "small" GAs rather than a single "big" GA in the tuning of the FIS.

5 Preliminary Results

The functions of this algorithm have been developed in MATLAB. For this computation, the physical parameters of the system have been set to the followings values: Mass of the free cart, (M_1), 0.2 [kg], mass of the rod, (M_2), 1 [kg], length of the rod, (L_2), 1 [m]. For the specified conditions, the tower's training process took 7 seconds. Unlike the traditional approach which training took more than 2 hours.

For the validation phase, in each case studied, a set of initial conditions have been considered; $\theta_0, \dot{\theta}_0, \ddot{\theta}_0, x_0, \dot{x}_0, \ddot{x}_0, F_0$. Both methodologies are able to keep the inverted pendulum upright in the operating range for which they have been trained. In addition, the two methods converge towards their stationary states in a similar way. Below are four cases (Fig 5. to Fig. 8) in which the pendulum control is more complex; where $\ddot{x}_0 < 0$ and $\ddot{\theta}_0 > 0$.

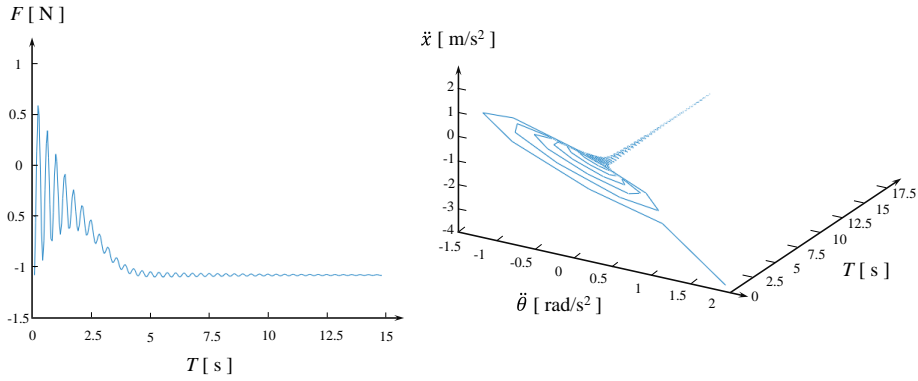


Fig. 5. Control of the IPFC for a given case 1. Force evolution in time (left). Linear and angular accelerations' convergence in time (right).

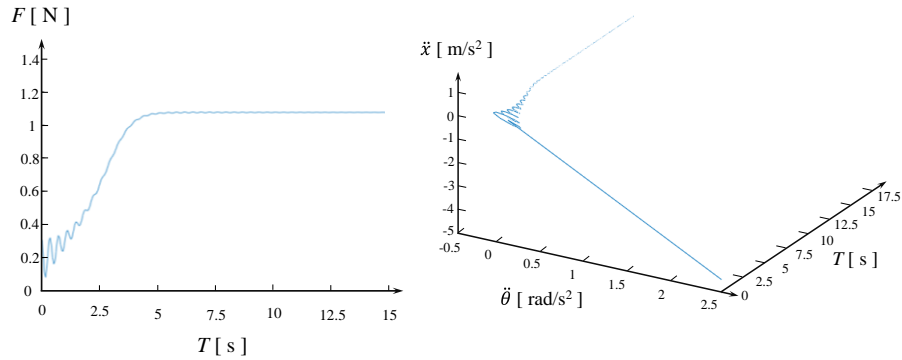


Fig. 6. Control of the IPFC for a given case 2. Force evolution in time (left). Linear and angular accelerations' convergence in time (right).

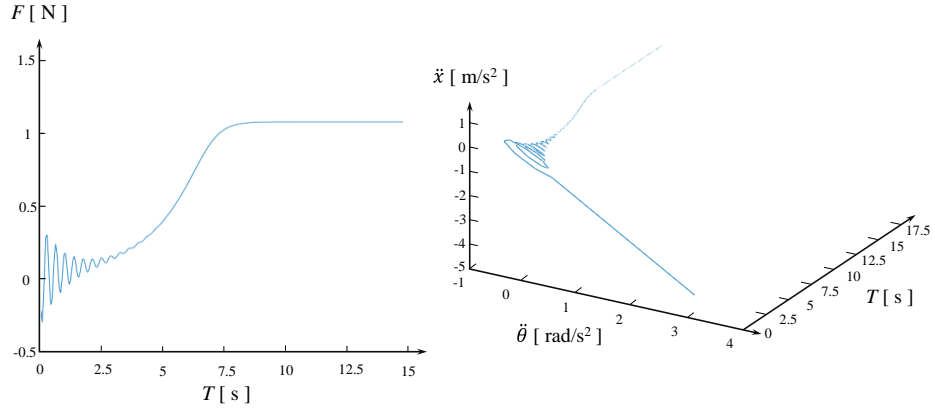


Fig. 7. Control of the IPFC for a given case 3. Force evolution in time (left). Linear and angular accelerations' convergence in time (right).

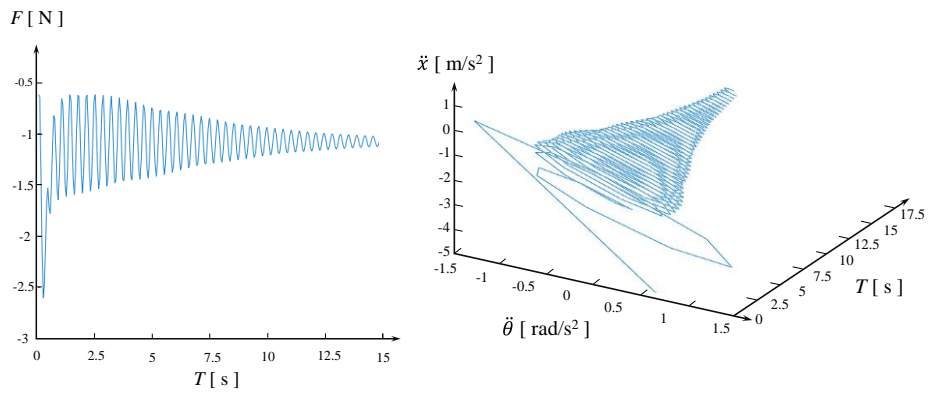


Fig. 8. Control of the IPFC for a given case 4. Force evolution in time (left). Linear and angular accelerations' convergence in time (right).

In addition to keeping the pendulum in an upright position, it is observed how the applied force is such that a tradeoff between $\dot{\theta}$ and \ddot{x} is obtained so that θ , $\dot{\theta}$ and \ddot{x} are minimal. These do not become exactly zero because of the granularity of the problem considered. Increasing the number of membership functions would make the values in the steady state closer to zero. On the other hand, it would also increase the computational cost. However, to verify the algorithm proposed in this study (which is the main objective) is sufficient.

6 Conclusions

A fast training algorithm for the control of symmetric systems has been proposed. Its effectiveness has been proven with the Inverted Pendulum + Free Cart system. The basis of the algorithm is a Fuzzy Inferencing System whose parameters have been optimized using different chained Genetic Algorithms. The training time is substantially improved by dividing the workspace into blocks and gradually transmitting the information obtained to the following phases of the optimization process. Using a single GA for the optimization of the entire system has a computational cost of more than 2 hours, whereas this method required less than 10 seconds. Furthermore, such benefit is achieved while still proving a successful and efficient result competitive with the traditional approach.

References

1. Ibañez, C.A., Frias, O.G., Castañón, M.S.: Lyapunov-Based Controller for the Inverted Pendulum Cart System. *Nonlinear Dynamics* 40, 367–374 (2005).
2. Belarbi, K., Titel, F.: Genetic algorithm for the design of a class of fuzzy controllers: an alternative approach. *IEEE Transactions on Fuzzy Systems* 8 (4), 398-405 (2000).
3. Sathyan, A., Cohen, K.: Development of a Genetic Fuzzy Controller and Its Application to a Noisy Inverted Double Pendulum. In: *Fuzzy Logic Based in Optimization Methods and Control Systems and Its Applications*. IntechOpen (2018).
4. Nasir, A. N. K., Ahmad, M. A., Rahmat, M. F.: Performance Comparison between LQR and PID Controllers for an Inverted Pendulum System. In: *AIP Conference Proceedings* 1052, 124 (2008).
5. Shiriaev, A., Pogromsky, A., Ludvigsen, H., Egeland, O.: On global properties of passivity-based control of an inverted pendulum. *International Journal of Robust and Nonlinear Control* 10, 283-300 (2000).
6. Kim, J., Moon Y., Zeigler, B. P.: Designing fuzzy net controllers using genetic algorithms. *IEEE Control Systems Magazine*, 15 (3), 66-72 (1995).
7. Hanafy, T. O. S.: Stabilization of inverted pendulum system using particle swarm optimization. In: *8th International Conference on Informatics and Systems (INFOS)*, pp. 207-210. IEEE, Cairo (2012).
8. Jacknoon, A., Abido, M. A.: Ant Colony based LQR and PID tuned parameters for controlling Inverted Pendulum. In: *International Conference on Communication, Control, Computing and Electronics Engineering (ICCCCEE)*, pp. 1-8. IEEE, Khartoum (2017).
9. Song, F., Smith, S. M.: A Takagi-Sugeno type fuzzy logic controller with only 3 rules for a 4 dimensional inverted pendulum system. In: *2000 IEEE International Conference on Systems, Man and Cybernetics Conference Proceeding*, pp. 3800-3805. IEEE, Nashville (2000).
10. Tatikonda, G. R. C., Battula, V. P., Kumar, V.: Control of inverted pendulum using adaptive neuro fuzzy inference structure (ANFIS). In: *Proceedings of 2010 IEEE International Symposium on Circuits and Systems*, pp. 1348-1351. Paris (2010).
11. Mamdani, E.H., Assilian, S.: An experiment in linguistic synthesis with a fuzzy logic controller. *International Journal of Man-Machine Studies* 7 (1), 1-13 (1975).